

# Pseudoscalar meson form factors and decays

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## Abstract

In this communication we discuss few topics related with modern experimental data on the physics of light pseudoscalar mesons. It includes the contribution of the pseudoscalar mesons to the muon anomalous magnetic moment (AMM),  $g-2$ , the rare decays of light pseudoscalar mesons to lepton pair, the transition form factors of pseudoscalar mesons at large momentum transfer, the pion transversity form factor.

Measuring the muon anomalous magnetic moment  $g-2$  and the rare decays of light pseudoscalar mesons into lepton pair  $P \rightarrow l^+l^-$  serve as important test of the standard model. To reduce the theoretical uncertainty in the standard model predictions the data on the transition form factors of light pseudoscalar mesons play significant role. Recently new data on behavior of these form factors at large momentum transfer was supplied by the BABAR collaboration. Within the nonlocal chiral quark model it shown how to describe these data and how the meson distribution amplitude evolves as a function of the dynamical quark masses and meson masses.

## I. INTRODUCTION

Among hadrons the pion plays very special role because it has the mass much lighter than other hadronic states. In theory it appears as a result of spontaneous breaking of chiral symmetry and gains its small mass due to small masses of nonstrange current quarks. It seems that confinement forces do not play any significant role in formation of the pion as a bound state.

Studies of the pion have long history and at the moment the processes with pion participation is one of standard hadronic background. From other side new highly-precise experiments related to rare decays or form factors at large momentum transfer serve as an important independent way to search for physics beyond the standard model and as test of the QCD dynamics. At the moment, there are some problems with the matching of the experimental data with the predictions of the SM. The most famous one is the discrepancy by three standard deviations between experiment [1] and SM theory [2, 3] for the muon anomalous magnetic moment (AMM). Another example is similarly large deviation between the recent precise experimental result on the rare  $\pi^0$  decay into  $e^-e^+$  pair [4] and the SM prediction [5].

## II. MUON G-2

The theoretical studies of the muon AMM  $g - 2$  (see for review [6–10]), the rare decays of light pseudoscalar mesons into lepton pairs [5, 11–14] and the comparison with the experimental results, offer an important low-energy tests of the SM.

The discrepancy between the present SM prediction of the muon AMM and its experimental determination [1] is  $(28.7 \pm 8.0) \cdot 10^{-10}$  ( $3.6\sigma$ ) [2]. The theoretical error is dominated by hadronic corrections. They are the vacuum polarization (HVP) and light-by-light (LbL) contributions. The latter can be estimated only in model dependent way. The latest results are the calculations of the pseudoscalar hadronic channel contribution within the instanton quark model [15] and the calculations within the so called [16] Dyson-Schwinger equation approach. In [15] in the full kinematic dependence of the meson-two-photon vertices from the virtualities of the mesons and photons is taken into account. It is demonstrated that the effect of the full kinematic dependence in the meson-photon vertices is to reduce the

contribution of pseudoscalar exchanges comparing with the most of previous estimates and the result is  $a_{\mu}^{\text{PS,LbL}} = (5.85 \pm 0.87) \cdot 10^{-10}$ .

The knowledge of the  $\eta(\prime)$  meson couplings to virtual photons is important for the calculation of the anomalous magnetic moment of the muon, being pseudoscalar exchange the major contribution to the hadronic light-by-light scattering. For illustration in Fig. 1 we present on the same plot the vertex  $F_{P^*\gamma\gamma}(p^2; 0, 0)$  in the timelike region  $p^2 \leq 0$  and the vertex  $F_{P^*\gamma\gamma}(p^2; p^2, 0)$  in the spacelike region  $p^2 \geq 0$  as they look in the instanton quark model (N $\chi$ QM) and vector meson dominance model (VMD). These two special kinematics match at zero virtuality  $p^2 = 0$ . The remarkable feature of this construction is that the first kinematics is connected with the decay of pseudoscalar mesons into two photons at physical points  $F_{P\gamma\gamma}(-M_M^2; 0, 0) = g_{P\gamma\gamma}$ , while the second kinematics is relevant for the light-by-light contribution to the muon AMM. Thus, the part of Fig. 1 at  $p^2 < 0$  describes the transition of the pion-two-photon vertex from the physical points of meson masses to the point with zero virtuality, which is the edge point of the interval where the integrand is defined. In VMD type of models there is no such dependence on the meson virtuality. Thus, the value of this vertex at zero meson virtuality is the same as the value of the vertex at the physical points of meson masses,  $F_{P\gamma\gamma}^{\text{VMD}}(p^2 = -M_{\eta,\eta'}^2; 0, 0) = F_{P\gamma\gamma}^{\text{VMD}}(0; 0, 0)$ . However, the  $\eta$  and  $\eta'$  mesons are much heavier than the pion and such extrapolation is too crude. One can see that for  $\eta$  and particularly for  $\eta'$  the difference between the values of the vertex at physical and zero virtuality points is large,  $F_{P\gamma\gamma}(p^2 = -M_{\eta,\eta'}^2; 0, 0) \gg F_{P\gamma\gamma}(0; 0, 0)$ . Thus, the contributions of the  $\eta$  and  $\eta'$  mesons to the muon AMM evaluated in N $\chi$ QM are strongly suppressed as compared with the VMD results that can only be considered as upper estimates of these contributions.

In [16] the hadronic LbL contribution to the muon AMM using the framework of Dyson-Schwinger equation (DSE) with the result  $a_{\mu}^{\text{PS,qLoop}} = (13.6 \pm 5.9) \cdot 10^{-10}$  that is bigger than most previous estimates. It is not easy to understand this result, in particular, in view of discussion in [17]. Moreover, it is instructive to compare the results on hadronic vacuum polarization contribution in the instanton [18] and DSE [19] based models. In [18] it was demonstrated that the dominant HVP contribution to muon AMM comes from the dynamical quark loop and contribution of the  $\rho$  meson is highly suppressed, while in [19] the opposite conclusion was made. It is still open question what model is more adequate in these calculations.

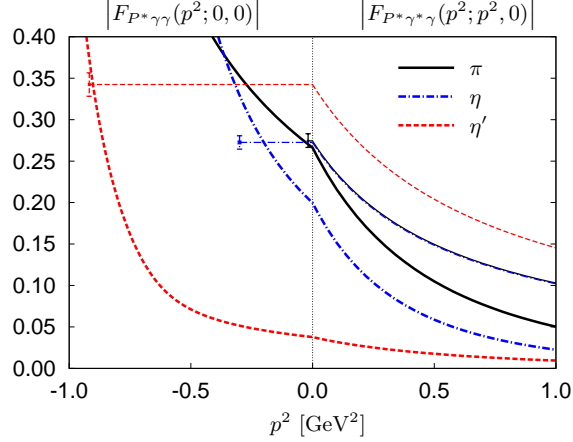


FIG. 1. Plots of the  $\pi^0, \eta$  and  $\eta'$  vertices  $F_{P^*\gamma\gamma}(p^2; 0, 0)$  in the timelike region and  $F_{P^*\gamma^*\gamma}(p^2; p^2, 0)$  in the spacelike region in N $\chi$ QM model (thick lines) and VMD model (thin lines). The points with error bars correspond to the physical points of the meson decays into two photons. The VMD curves for  $\pi^0$  and  $\eta$  are almost indistinguishable.

### III. RARE PION DECAY $\pi^0 \rightarrow e^+e^-$

The situation with the rare decays of the light pseudoscalar mesons into lepton pairs became intriguing after the KTeV E799-II experiment at FermiLab [4] in which the pion decay into an electron-positron pair was measured with high accuracy ( $R(P \rightarrow l^+l^-) = \Gamma(P \rightarrow l^+l^-) / \Gamma_{tot}$ )

$$R^{\text{KTeV}}(\pi^0 \rightarrow e^+e^-) = (7.49 \pm 0.38) \cdot 10^{-8}. \quad (1)$$

The standard model prediction gives [5, 13]

$$R^{\text{Theor}}(\pi^0 \rightarrow e^+e^-) = (6.2 \pm 0.1) \cdot 10^{-8}, \quad (2)$$

which is  $3.1\sigma$  below the KTeV result (1). Other pseudoscalar mesons rare decays into lepton pair are also well suited as a test of the standard model (see Table). From experimental point of view the most interesting are the decays of  $\eta$  and  $\eta'$  to muon pair. The branchings for these processes theoretically bounded as from below as well from the above [5, 13]. Further independent experiments for  $\pi^0 \rightarrow e^+e^-$  at WASAatCOSY [20] and for  $\eta(\eta' + l^-)$  KLOE [21] and BES III [22] and other facilities will be crucial for resolution of the problem with the rare leptonic decays of light pseudoscalar mesons. More precise data on the transition

form factors in wider region of momentum transfer are expected soon from the BABAR, BELLE (at large momentum transfer) and KEDR, KLOE [23] (at small momentum transfer) collaborations. These data would allow to make more accurate theoretical predictions.

TABLE I. Values of the branchings  $R(P \rightarrow l^+l^-) = \Gamma(P \rightarrow l^+l^-)/\Gamma_{tot}$  obtained in approach [5, 13] and compared with the available experimental results.

$R$	Unitary bound	CLEO+BABAR bound	CLEO+BABAR +OPE	With mass corrections	Experiment
$R(\pi^0 \rightarrow e^+e^-) \times 10^8$	$\geq 4.69$	$\geq 5.85 \pm 0.03$	$6.23 \pm 0.12$	6.26	$7.49 \pm 0.38$ [4]
$R(\eta \rightarrow \mu^+\mu^-) \times 10^6$	$\geq 4.36$	$\leq 6.60 \pm 0.12$	$5.35 \pm 0.27$	4.76	$5.8 \pm 0.8$ [24, 25]
$R(\eta \rightarrow e^+e^-) \times 10^9$	$\geq 1.78$	$\geq 4.27 \pm 0.02$	$4.53 \pm 0.09$	5.19	$\leq 2.7 \cdot 10^4$ [26]
$R(\eta' \rightarrow \mu^+\mu^-) \times 10^7$	$\geq 1.35$	$\leq 1.44 \pm 0.01$	$1.364 \pm 0.010$	1.24	
$R(\eta' \rightarrow e^+e^-) \times 10^{10}$	$\geq 0.36$	$\geq 1.121 \pm 0.004$	$1.182 \pm 0.014$	1.83	

There are quite few attempts in the literature, to explain the excess of the experimental data on the  $\pi^0 \rightarrow e^+e^-$  decay over the SM prediction, as a manifestation of physics beyond the SM. In Ref. [27], it was shown that this excess could be explained within the currently popular model of light dark matter involving a low mass ( $\sim 10$  MeV) vector bosons  $U_\mu$ , which presumably couple to the axial-vector currents of quarks and leptons. Another possibility was proposed in Ref. [28, 29], interpreting the same experimental effect as the contribution of the light CP-odd Higgs boson appearing in the next-to-minimal supersymmetric SM. The latter version perhaps excluded by modern experiments.

#### IV. PSEUDOSCALAR MESON TRANSITION FORM FACTORS

The main limitation on realistic predictions for above processes originates from the large distance contributions of the strong sector of the SM, where perturbative QCD does not work. In order to diminish the theoretical uncertainties, the use of the experimental data on the pion charge and transition form factors are of crucial importance. The first one, measured in  $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$  by CMD-2 [30], SND [31], KLOE [32], and BABAR [33] provides an estimate for the hadron vacuum polarization contribution to muon  $g-2$ , with accuracy better than 1%. The second one, measured in  $e^+e^- \rightarrow e^+e^-P$  for spacelike photons

by CELLO [34], CLEO [35], and BABAR [36–38] collaborations and in  $e^+e^- \rightarrow P\gamma$  for timelike photons by the BABAR [39] collaboration, is essential to reduce the theoretical uncertainties in the estimates of the contributions of the hadronic light-by-light process to the muon  $g - 2$  and in the estimates of the decay widths of  $P \rightarrow l^+l^-$ . The BABAR data [36, 39] on the large momentum behavior of the form factors cause the following problems for their theoretical interpretation [40, 41]: 1) An unexpectedly slow decrease of the pion transition form factor at high momenta [36], 2) the qualitative difference in the behavior of the pion and  $\eta, \eta'$  form factors at high momenta [37], 3) inconsistency of the measured ratio of the  $\eta, \eta'$  form factors with the predicted one [39].

In Figs. 2, 3, 4 and 5 the data for the  $\pi^0$ ,  $\eta$ ,  $\eta'$  and  $\eta_c$  transition form factors from the CELLO, CLEO, and BABAR collaborations are presented. In Figs. 3 and 4, the CLEO [42] and BABAR [39] points, measured in the timelike region, are drawn at  $Q^2 = 14.2 \text{ GeV}^2$  and  $Q^2 = 112 \text{ GeV}^2$ , correspondingly, assuming that the spacelike and timelike asymptotics of the form factor are equal. It is seen from the Figs. 3 and 4, that the spacelike and timelike points are well conjugated.

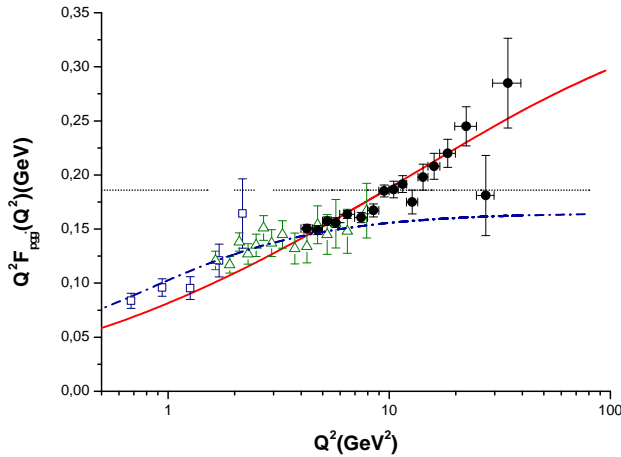


FIG. 2. The transition form factor  $\gamma^*\gamma \rightarrow \pi^0$ . The data are from the CELLO [34], CLEO [35] and BABAR [36] Collaborations. The solid line is the nonlocal chiral quark model calculations and the dash-dot line is the parametrization (6). The dotted line is massless QCD asymptotic limit.

At zero momentum transfer, the transition form factor is fixed by the two-photon decay width (see for recent discussion [43])

$$F_{P\gamma\gamma}^2(0,0) = \frac{1}{(4\pi\alpha)^2} \frac{64\pi\Gamma(P \rightarrow \gamma\gamma)}{M_P^3}, \quad (3)$$

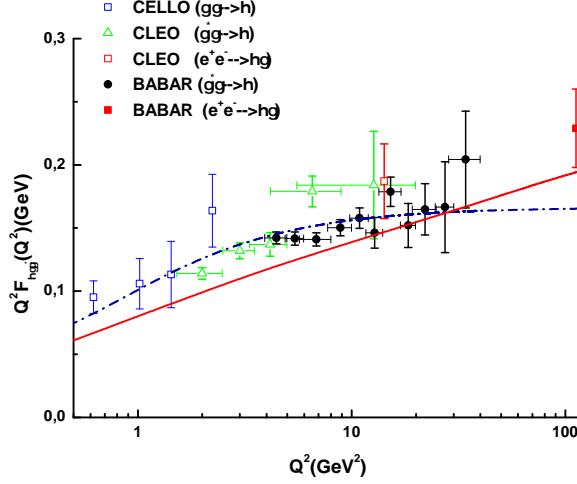


FIG. 3. The transition form factor  $\gamma^*\gamma \rightarrow \eta$ . The data are from the CELLO [34], CLEO [35] and BABAR [37] Collaborations. The CLEO results obtained in different  $\eta$  decay modes are averaged. The CLEO and BABAR points, measured in the timelike region  $\gamma^* \rightarrow \eta\gamma$  [42] and [39], are drawn at  $Q^2 = 14.2 \text{ GeV}^2$  and  $Q^2 = 112 \text{ GeV}^2$ , correspondingly, assuming that the spacelike and timelike asymptotics of the form factor are similar. The solid line is the nonlocal chiral quark model calculations and the dash-dot line is the parametrization (6).

where  $\alpha$  is the QED coupling constant,  $M_P$  is the resonance mass and  $\Gamma(P \rightarrow \gamma\gamma)$  is the two-photon partial width of the meson  $P$ . The axial anomaly predicts

$$F_{P\gamma\gamma^*}(Q^2 = 0, 0) \approx \frac{1}{4\pi^2 f_P}, \quad (4)$$

where  $f_P$  is the meson decay constant. Under assumption of factorization, perturbative QCD predicts the asymptotic behavior of the  $F_{P\gamma\gamma^*}^2(Q^2, 0)$  transition form factors as  $Q^2 \rightarrow \infty$  [44]

$$F_{P\gamma\gamma^*}(Q^2 \rightarrow \infty, 0) \sim \frac{2f_P}{Q^2}. \quad (5)$$

The perturbative QCD corrections to this expression at large momentum transfer are quite small [45–48]. The CLEO (and CELLO) collaboration parameterized their data by a formula similar to that proposed by Brodsky and Lepage in [44], but with the pole mass being a free fitting parameter [35],

$$F_{\pi\gamma\gamma^*}^{\text{CLEO}}(Q^2, 0) = \frac{1}{4\pi^2 f_P} \frac{1}{1 + Q^2/\Lambda_P^2}, \quad (6)$$

where the values of  $f_P$  are estimated from (3) and (4) [35]:  $f_\pi = 92.3 \text{ MeV}$ ,  $f_\eta = 97.5 \text{ MeV}$ ,  $f_{\eta'} = 74.4 \text{ MeV}$ . In Figs. 2, 3, 4 and 5 the parametrizations (6) with parameters  $\Lambda_\pi = 776 \text{ MeV}$ ,  $\Lambda_\eta = 800 \text{ MeV}$ ,  $\Lambda_{\eta'} = 859 \text{ MeV}$  and  $\Lambda_{\eta_C} = 2.92 \text{ GeV}$  are shown by dot-dashed lines.

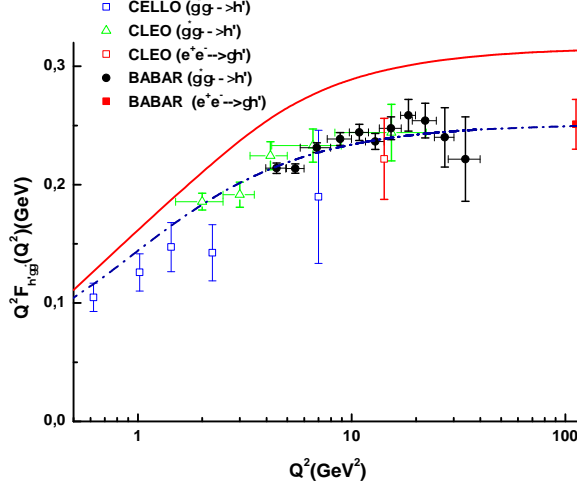


FIG. 4. The transition form factor  $\gamma^* \gamma \rightarrow \eta'$ . The data are from the CELLO [34], CLEO [35] and BABAR [37] Collaborations. The CLEO results obtained in different  $\eta'$  decay modes are averaged. The CLEO and BABAR points, measured in the timelike region  $\gamma^* \rightarrow \eta \gamma$  [42] and [39], are drawn at  $Q^2 = 14.2 \text{ GeV}^2$  and  $Q^2 = 112 \text{ GeV}^2$ , correspondingly, assuming that the spacelike and timelike asymptotics of the form factor are similar. The solid line is the nonlocal chiral quark model calculations and the dash-dot line is the parametrization (6).

We see that the parametrization (6) describes the  $\eta'$  and  $\eta_C$  mesons form factors (Figs. 4, 5) well at all measured momentum transfer. However, it starts to deviate from data for the pion and probably for the eta mesons form factors at momentum transfers squared larger than  $10 \text{ GeV}^2$  (Figs. 2, 3). Namely, the high momentum transfer data obtained by BABAR collaboration show evident growth of the form factor multiplied by  $Q^2$  at large  $Q^2$  for the pion and probably for the eta. This is in contradiction with asymptotic formula (5) and unexpected from the QCD factorization approach [48]<sup>1</sup>. The  $\eta_c$  form factor is in good agreement with predictions [49].

In [50, 51] the asymptotic of the pseudoscalar meson transition form factor generalizing the result (5) was derived

$$F_{P\gamma^*\gamma}(0; Q^2 \rightarrow \infty, 0) = \frac{2f_P}{3} \int_0^1 dx \frac{\varphi_P^f(x)}{D(xQ^2)}, \quad (7)$$

where  $D(k^2)$  is the nonperturbative inverse quark propagator with property  $D(k^2 \rightarrow \infty) \rightarrow k^2$  and  $\varphi_\pi(x)$  is the meson distribution amplitude (DA) given in the nonlocal model by (see

<sup>1</sup> Small radiative corrections can be attributed to slight changes in the parameters  $\Lambda_P$  in (6).



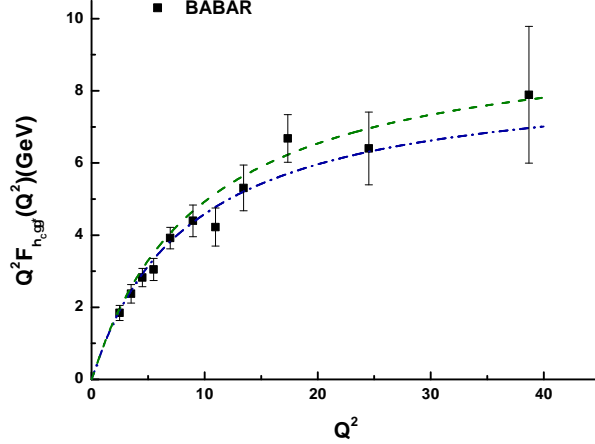


FIG. 5. The transition form factor  $\gamma^* \gamma \rightarrow \eta_c$ . The data are from the BABAR [38] Collaboration. The dashed line is the nonlocal model calculation and the dot-dashed line is the fit. (The normalization is arbitrary)

for notations and the  $\alpha$  representation [50, 51])

$$\varphi_P(x) = \frac{N_c}{4\pi^2 f_{\text{PS},\pi}^2} \int_0^\infty \frac{dL}{L} e^{x\bar{x}LM_P^2} (xG_{m,0}(xL, \bar{x}L) + \bar{x}G_{0,m}(xL, \bar{x}L)), \quad (8)$$

with  $\bar{x} = (1 - x)$  and

$$\int_0^1 dx \varphi_P(x) = 1.$$

The properties of the form factor at large  $Q^2$  strongly depend on the end point behavior of the meson DA. In [52–54] it was assumed that the pion is almost pointlike and that its DA can be almost constant (flat) in its shape. In this case the integrand in (7) is poorly convergent and the  $1/Q^2$  behavior of the form factor is enhanced by logarithmic factor  $\ln(Q^2/M_q^2)$  asymptotically or in rather wide region of large  $Q^2$ , where  $M_q$  is dynamical (constituent) quark mass. At the same time the DA with suppressed end point behavior lead to the standard result (6).

The solid line in Fig. 2 is the pion transition form factor calculated in the nonlocal chiral quark model [50, 51] with the mass parameters  $M_Q = 135$  MeV and strange quark mass  $M_s = 250$  well describes the BABAR data. There are other attempts to explain the BABAR data on the pion form factor within different factorization schemes [55–61].

By using singlet-octet mixing scheme one has for  $\eta$  and  $\eta'$  meson form factors [15]

$$\begin{aligned}
F_{\eta\gamma^*\gamma}(m_\eta^2; Q^2, 0) &= \frac{1}{3\sqrt{3}} \left[ (5F_u(m_\eta^2; Q^2, 0) - 2F_s(m_\eta^2; Q^2, 0)) \cos \theta(m_\eta^2) - \right. \\
&\quad \left. - \sqrt{2} (5F_u(m_\eta^2; Q^2, 0) + F_s(m_\eta^2; Q^2, 0)) \sin \theta(m_\eta^2) \right], \\
F_{\eta'\gamma^*\gamma}(m_{\eta'}^2; Q^2, 0) &= \frac{1}{3\sqrt{3}} \left[ (5F_u(m_{\eta'}^2; Q^2, 0) - 2F_s(m_{\eta'}^2; Q^2, 0)) \sin \theta(m_{\eta'}^2) + \right. \\
&\quad \left. + \sqrt{2} (5F_u(m_{\eta'}^2; Q^2, 0) + F_s(m_{\eta'}^2; Q^2, 0)) \cos \theta(m_{\eta'}^2) \right], \tag{9}
\end{aligned}$$

From these expressions it is clear that in general it is not possible to separate the nonstrange and strange components of the form factors as it was attempted to do in [37] and then used in some theoretical works.

The solid line in Figs. 3, 4, is the eta and eta prime transition form factors calculated in the nonlocal chiral quark model [50, 51] with the mass parameters  $M_Q = 135$  MeV and strange quark mass  $M_s = 250$  well describes the BABAR data.

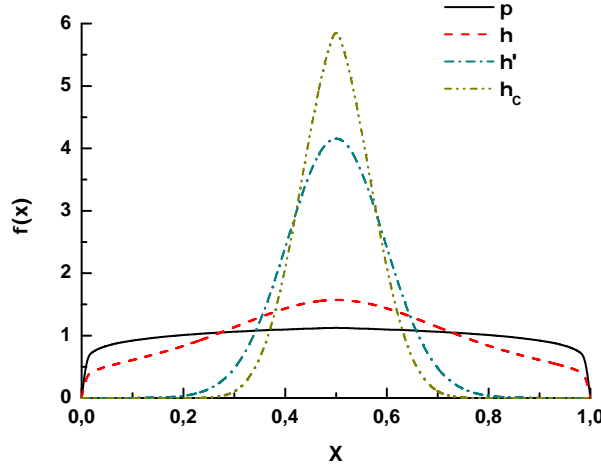


FIG. 6. The pion,  $\eta$ ,  $\eta'$ , and  $\eta_c$  distribution amplitudes for the nonlocal chiral quark model with parameters  $M_u = 135$  MeV,  $M_s = 250$  MeV,  $M_c = 1550$  MeV,

From above results one may conclude that the possible origin of the difference of the asymptotic behavior of the pion and  $\eta$  from one side and of the  $\eta'$  meson form factors is the fact that  $\eta'$  is much heavier than the pion and  $\eta$ . We have checked that it is this property responsible for the change of the almost flat DA in the pion and  $\eta$  cases for the suppressed at end points DA in the cases of the  $\eta'$ ,  $\eta_c$  mesons (Fig. 6) [65].

## V. PION TRANSVERSITY FORM FACTORS

The *transversity* form factors (TFFs) of the pion provide valuable insight into chirally-odd generalized parton distribution functions (GPDs) as well as into the non-trivial spin structure of the pion. These interesting quantities have been determined for the first time on the lattice [62]. Formally, the TFFs, denoted as  $B_{Tni}^\pi(t)$ , are defined as

$$\langle \pi^+(P') | \mathcal{O}_T^{\mu\nu\mu_1\cdots\mu_{n-1}} | \pi^+(P) \rangle = \mathcal{AS} \bar{P}^\mu \Delta^\nu \sum_{i=0, \text{even}}^{n-1} \Delta^{\mu_1} \cdots \Delta^{\mu_i} \bar{P}^{\mu_{i+1}} \cdots \bar{P}^{\mu_{n-1}} \frac{B_{Tni}^{\pi,u}(t)}{m_\pi}, \quad (10)$$

where  $P'$  and  $P$  are the momenta of the pion, the  $\bar{P} = \frac{1}{2}(P' + P)$ ,  $\Delta = P' - P$ , and  $t = \Delta^2$ . The symbol  $\mathcal{AS}$  denotes symmetrization in  $\nu, \dots, \mu_{n-1}$ , followed by antisymmetrization in  $\mu, \nu$ , with the additional prescription that traces in all index pairs are subtracted. The tensor operators are given by

$$\mathcal{O}_T^{\mu\nu\mu_1\cdots\mu_{n-1}} = \mathcal{AS} \bar{u}(0) i\sigma^{\mu\nu} iD^{\mu_1} \cdots iD^{\mu_{n-1}} u(0),$$

where  $D = \frac{1}{2}(\vec{D} - \overleftarrow{D})$  denoting the QCD covariant derivative.

The available full-QCD lattice results [62] are for  $B_{10}^{\pi,u}$  and  $B_{20}^{\pi,u}$  and for  $-t$  reaching 2.5 GeV<sup>2</sup>, with moderately low values of the pion mass  $m_\pi \sim 600$  MeV. The calculation uses the set of QCDSF/UKQCD  $N_f = 2$  improved Wilson fermion and the Wilson gauge-action ensembles.

Within the effective nonlocal model one gets from the triangle diagram corresponding to (10) the transversity pion form factors (see [63, 64], where notations are explained)

$$B_{Tni}^u(t) = \frac{N_c}{4\pi^2 f_\pi^2} \frac{(n-1)!}{i!(n-1-i)!} \int \frac{d(\alpha\beta\gamma)}{\Delta^{n+2}} \beta^{n-1-i} \left( \frac{\gamma-\alpha}{2} \right)^i e^{-\frac{\alpha\gamma}{\Delta}t} [2\alpha G_{m,0,0}(\alpha, \beta, \gamma) + \beta G_{0,m,0}(\alpha, \beta, \gamma)], \quad (11)$$

where  $i = 0, 2, \dots \leq n-1$  and pion transversity generalized parton distribution (Fig. 7)

$$E_T^\pi(X, \xi, t) = \Theta(X + \xi) \frac{N_c}{4\pi^2 f_\pi^2} \int_0^\infty d\gamma \int_{\max\{0, \gamma \frac{\xi-X}{\xi+X}\}}^\infty d\alpha e^{-\frac{\alpha\gamma}{\Delta}t} \frac{\alpha G_{m,0,0}(\alpha, \beta, \gamma) + \beta G_{0,m,0}(\alpha, \beta, \gamma) + \gamma G_{0,0,m}(\alpha, \beta, \gamma)}{\Delta^2(1-X)}, \quad (12)$$

where  $\xi = -\frac{(aq)}{2(aP)}$ , ( $a^2 = 0$ ),  $\beta = \frac{(X+\xi)\alpha+(X-\xi)\gamma}{1-X}$ ,  $\Delta = [\alpha + \gamma + \xi(\alpha - \gamma)]/(1-X)$ . These studies show how the spinless pion acquires a non-trivial spin structure within the framework of chiral quark models.

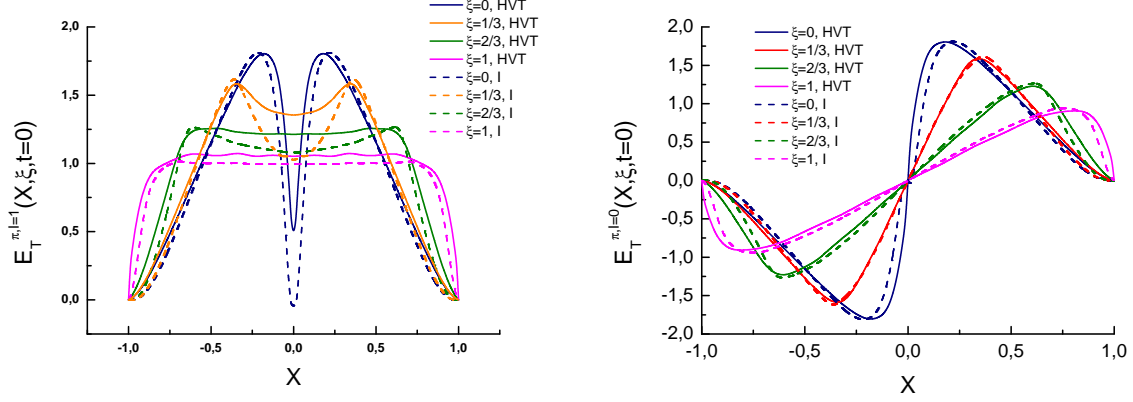


FIG. 7. The pion tGPD for isovector and isoscalar cases in the HVT model (solid lines) and in the instanton model (dashed lines) for several values of  $\xi$ .

## VI. CONCLUSIONS

The main conclusion is that the study of pion and other pseudoscalar mesons provides reach information about dynamics of strong interactions and in some rare processes testifies the standard model. New high statistics experiments in wider kinematical region are urgent for further progress. It is expected new interesting theoretical results of the nonperturbative QCD dynamics from the Lattice QCD and effective chiral quark models.

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